

BLACK HOLE MASS DETERMINATIONS FROM ORBIT SUPERPOSITION MODELS ARE RELIABLE

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ABSTRACT

We show that orbit-superposition dynamical models (Schwarzschild's method) provide reliable estimates of nuclear black hole masses and errors when constructed from adequate orbit libraries and kinematic data. We thus rebut two recent papers that argue that BH masses obtained from this method are unreliable. These papers claim to demonstrate that the range of allowable BH masses derived from a given dataset is artificially too narrow as a result of an inadequate number of orbits in the library used to construct dynamical models. This is an elementary error that is easily avoided. We describe a method to estimate the number and nature of orbits needed for the library. We provide an example that shows that this prescription is adequate, in the sense that the range of allowable BH masses is not artificially narrowed by use of too few orbits. This is illustrated by showing that the χ^2 versus BH-mass curve does not change beyond a certain point as more orbits are added to the library. At that point, the phase-space coverage of the orbit library is good enough to estimate the BH mass, and the χ^2 profile provides a reliable estimate of its errors.

A second point raised by critics is that kinematic data are generally obtained with insufficient spatial resolution (compared to the BH radius of influence) to obtain a reliable mass. We make the distinction between *unreliable* determinations and *imprecise* ones. We show that there are several different properties of a kinematic dataset that can lead to *imprecise* BH determinations (insufficient resolution among them), but none of the attributes we have investigated leads to an unreliable determination. In short, the degree to which the BH radius of influence is resolved by spectroscopic observations is already reflected in the BH-mass error envelope, and is not a hidden source of error. The BH masses published by our group and the Leiden group are reliable.

Subject headings: galaxies: nuclei — galaxies: statistics — galaxies: general

1. INTRODUCTION

There are almost twenty detections of massive black holes (BHs) in galaxy centers that employ the technique of orbit superposition modeling (van der Marel *et al.* 1998; Cretton & van den Bosch 1999; Cretton *et al.* 1999; Gebhardt *et al.* 2000a, 2003; Cappellari *et al.* 2002; Verolme *et al.* 2002b). The orbit superposition technique is based on a method originally invented by Schwarzschild (1979), who noted that the time-averaged orbits in a stationary potential can be summed (at finite spatial resolution) to match the mass distribution that gives rise to the potential, thereby producing an equilibrium dynamical model. The Schwarzschild method was first used to analyze kinematic data for evidence of BHs in galaxy centers by Richstone & Tremaine (1985) for M87 and Dressler & Richstone (1988) for M31 and M32. Those early models were spherically symmetric. In modern implementations of this method, large sets of orbits are run in a specified axisymmetric potential (based on the starlight distribution and a central point mass) and then a non-negative linear superposition of orbits is found

that best matches the kinematic and photometric observations. The usual goal of this process is the determination of only two quantities, the stellar mass-to-light ratio Υ (assumed to be independent of position) and the BH mass M_\bullet .

Valluri, Merritt & Emsellem (2004) (hereafter VME) and Cretton & Emsellem (2004) have challenged the reliability of BH mass determinations obtained via orbit superposition modeling. Cretton & Emsellem also argue that the uncertainties noted by VME can be dealt with via regularization (smoothing the distribution of orbit weights).

VME raise a long list of problems and features of the orbit-superposition analysis technique. They focus on two main issues: orbit insufficiency and data insufficiency. First, they contend that the BH mass determination is artificially narrowed through the use of too small an orbit library and imply, but do not show, that we have made this mistake. In addition they suggest that the absence of a flat-bottomed χ^2 profile as a function of BH mass indicates that this problem has occurred. Second, they suggest that the BH mass M_\bullet can only be determined if the radius of influence $r_i = GM_\bullet/\sigma^2$ where the BH dominates

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the stellar dynamics is well-resolved spectroscopically (here σ is the line-of-sight velocity dispersion). In each of their main points they have fastened on one facet of rather complex problems.

We show in §2 that the use of an orbit library which is too small is an easily avoided elementary trap. In §3 we refer the reader to other work in which we show that we have adequate resolution to determine BH masses reliably, and argue that the presence of extensive radial kinematic coverage mitigates the uncertainties associated with a “not well-resolved” radius of influence. We also show that the “flat-bottomed” behavior of χ^2 is a feature whose presence depends on several features of the quality and extent of the kinematic data, not just the size of the orbit library.

The principal result is that a χ^2 profile constructed using our method does indicate the range of acceptable BH masses in a manner easily judged by the readers of our papers.

2. ENOUGH IS ENOUGH: ORBIT SUFFICIENCY

There appear to be at least three independent axisymmetric orbit-superposition codes, one developed by various Leiden students starting with van der Marel and continuing through Verolme and Cappellari, another developed by Gebhardt and Richstone and used by our collaboration (the “nukers”) since 1996, and a third described by VME. All practitioners agree that the orbits can be sampled by examining launch points in the three-dimensional space spanned by the isolating integrals: the energy per unit mass E , the z -component of the angular momentum l_z , and the third integral I_3 . We further agree that this space is finite—only bound orbits matter ($E < 0$), the range of l_z at fixed energy lies between zero and the angular momentum of a circular orbit of that energy, and the distribution of the third integral can be sampled by launching the orbits with zero velocity from positions spread across the zero velocity surface at fixed E and l_z . The remaining question is how densely or sparsely spaced these orbital parameters need to be (and therefore how many orbits are required). The answer depends in part on the spatial resolution sought in the model, and in part on the complexity of the orbit structure in phase space, and therefore on the mass distribution.

Nonetheless, it is possible to estimate the number of orbits required to represent all possibilities. The eventual construction of the model requires matching the (stellar) mass density in a set of bins spanning the physical space of the (axisymmetric) model. This bin set is composed of one radial dimension (of n_r bins, which are logarithmically spaced except very near the center) and one angular dimension (of n_β bins, which are equally spaced in $\sin(\beta)$ where β is the latitude). To fully sample the equatorial orbits, we follow orbits with apocenter and pericenter in all possible pairs of radial bins; this requires roughly $n_r^2/2$ orbits. We then assign E and l_z from the apocenter sampling of the equatorial orbits and then use the same E and l_z for non-equatorial orbits. For each E and l_z pair, we drop a set of orbits from points on the zero-velocity curve equally spaced in $\sin(\beta)$ and more finely spaced than the angular bins. The models described here have 20 radial bins and 5 angular bins, hence our *nominal* orbit library contains $N_{orb} = 2 \times n_r^2/2 \times f_\beta \times n_\beta = 2 \times 20^2/2 \times 25 = 10,000$ orbits. The initial factor of 2 comes from including each orbit’s retrograde mirror image in the library, and the trailing factor of 25 results from oversampling the 5 angular bins by a factor of $f_\beta = 5$.

Even if one does not trust this estimate, it is straightfor-

ward to investigate whether the orbit library is large enough by changing the number of orbits in the library. If the results are independent of the number of orbits then presumably the library is large enough.

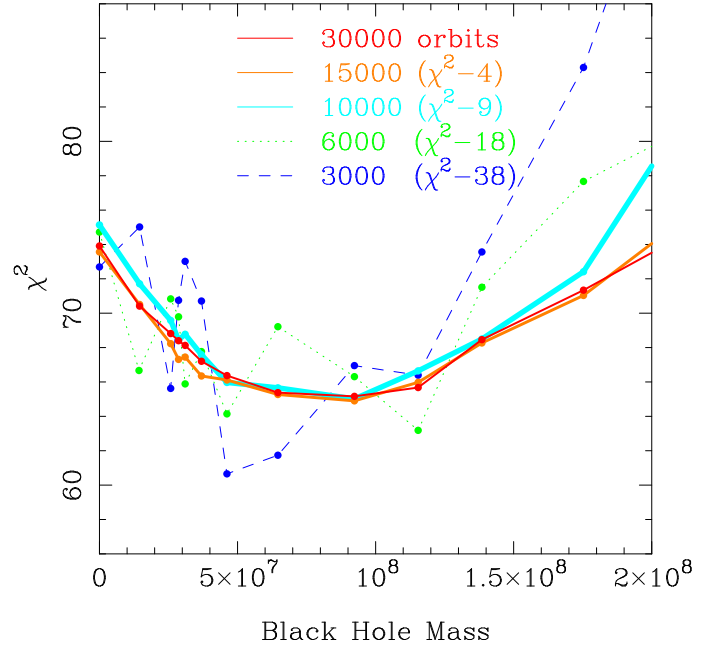


FIG. 1.— The χ^2 goodness of fit obtained by comparing model kinematics to observed kinematic data for NGC 891, versus model black hole mass. The models were constructed with successively finer coverage of phase space (and therefore larger numbers of orbits in the library), as described in the text. Once the number of orbits (in this example) exceeds 10,000 the χ^2 profile is independent of the size of the library, except for small vertical offsets which we have removed for plotting purposes. In all these experiments we used the kinematic dataset described in §3.

We created an orbit library as described above to model the galaxy NGC 821 and illustrate the results in Figure 1. The observations are described in Pinkney *et al.* (2003) and in §3 and the modeling is described in Gebhardt *et al.* (2003). We chose NGC 821 to correct an error in the models in Gebhardt *et al.* (2003), which used an incorrect Hubble Space Telescope (HST) point-spread function that did not reflect the spatial binning of the STIS CCD (this was the only one of the 12 galaxies in that paper in which this error was made). The revised mass is $(8.5 \pm 3.5) \times 10^7 M_\odot$, at a distance of 24.1 megaparsecs, a factor of 2.3 higher than was reported previously (at the same distance). In order to conduct the experiment illustrated in Figure 1 we oversampled the grid by a factor of 2 relative to the prescription above in each dimension and then dropped out orbits at random to obtain the libraries illustrated. In Figure 1 we show the χ^2 profile as a function of BH mass (we have already marginalized over the galaxy mass-to-light ratio).

For small orbit libraries, Figure 1 exhibits the ragged χ^2 emphasized by VME. It also shows that for our nominal library with $N_{orb} = 10,000$ orbits, the χ^2 profile is smooth enough to infer BH mass estimates and uncertainties, and that that larger libraries do not alter the mass estimate. Once the number of orbits is large enough that the χ^2 profile does not change as it is increased further, *the inferred BH mass does not depend on the number of orbits*, and the phase-space coverage *must* be good enough.

3. MORE IS BETTER: DATA SUFFICIENCY

A second major point made by VME is that estimates of BH mass are unreliable if the BH radius of influence r_i is not “well-resolved” by the kinematic data. This is one facet of the rich question of what different kinds of kinematic data reveal about the gravitational field of the galaxy. The geometry of the kinematic data can be characterized by: (1) the spatial resolution compared to r_i ; (2) the radial extent of the data (how far out the data extend); (3) the angular coverage (how many position angles, or whether there is integral-field data); and (4) the sparseness of the data (in radius and angle). In addition to these geometrical characteristics, the quality of the data (signal-to-noise and systematic errors such as template mismatches) also determines what one can measure. A complete investigation of all these issues is beyond the scope of this paper. We emphasize the distinction between *precision* (are the error bars small?) and *reliability* (are the error bars accurately estimated?). We shall argue that geometric limitations to the quality of the kinematic data of the four kinds listed above reduce the precision of estimates of M_\bullet , but the range of acceptable masses can still be judged from the χ^2 profile, and therefore the mass determination is reliable.

We parameterize the resolution of the kinematic data near the center in terms of $\aleph = r_i/\theta D$, where r_i is the BH radius of influence defined earlier, D is the distance, and θ is the full-width-half-maximum telescope resolution. VME argue that the resolution \aleph must be much greater than unity for accurate BH mass determinations, although they do not give a clear numerical criterion. A number of the BH mass determinations have \aleph only slightly larger than unity. In the example of NGC 821 discussed below, $\theta \sim 0''.08$ (the observations were made at 8500\AA with a $0''.1$ slit), $D = 24.1\text{ pc}$ and $r_i \sim 8\text{ pc}$, so $\aleph \sim 0.9$.

VME’s argument that observations with this resolution cannot determine M_\bullet is made without regard to the signal-to-noise of the data available. Even in the limit $\aleph = 0$, excellent S/N data can reveal the presence of a BH. Given a model with constant mass-to-light ratio Υ , a central mass M_\bullet and a perfect LOSVD measured with $\aleph \ll 1$, the shape of the LOSVD at low velocities (comparable to velocity dispersion of the galaxy as a whole) determines Υ , and the shape and extent of the high-velocity wings of the LOSVD determines the mass of a central BH.

We have investigated the effect of varying the resolution \aleph on our determinations of M_\bullet by comparing our masses determined using both high-resolution HST data and low-resolution ground-based data to masses determined using the same method with ground-based data alone. The addition of HST data generally improves \aleph by a factor of 5, from less than unity to greater than unity. The results of these experiments are shown in Figure 8 in Gebhardt *et al.* (2003) and Figure 4 in Kormendy (2004). We find that improved resolution generally improves the precision of the measured M_\bullet by narrowing the χ^2 profile, but the improved best-fit M_\bullet always lies within the range given by the estimated errors from the low-resolution data. This experiment is evidence that the χ^2 profiles do exactly what they are supposed to—they provide an estimate of M_\bullet and of its uncertainty. Lower resolution data yield less precise values of M_\bullet , but not unreliable ones.

We next investigate the influence of kinematic data at larger

radii on the precision of the BH measurement (points 2 and 3 above), using data from NGC 821 (see Figure 2). This example¹² shows that the χ^2 profile broadens and flattens as data at large radii and along the minor axis are discarded. The changes in the χ^2 profile imply a decrease in the precision of the measurement of M_\bullet and indicate the increased errors. If only HST data is used then models with and without a BH are both acceptable, because of the degeneracy between M_\bullet and Υ when only data inside and near the radius of influence are used. Note that the one model with only major axis data (the dashed line) yields a flat-bottomed χ^2 profile and a very uncertain estimate of M_\bullet compared to the model with additional data along the minor axis. We conclude that inadequate spatial coverage of kinematics far from the BH can lead to a poorly defined BH mass and a flat-bottomed χ^2 just as readily as can poorly resolved data ($\aleph > 1$) in the galaxy center. Again, inadequate spatial coverage leads to imprecise BH mass estimates, but not unreliable ones.

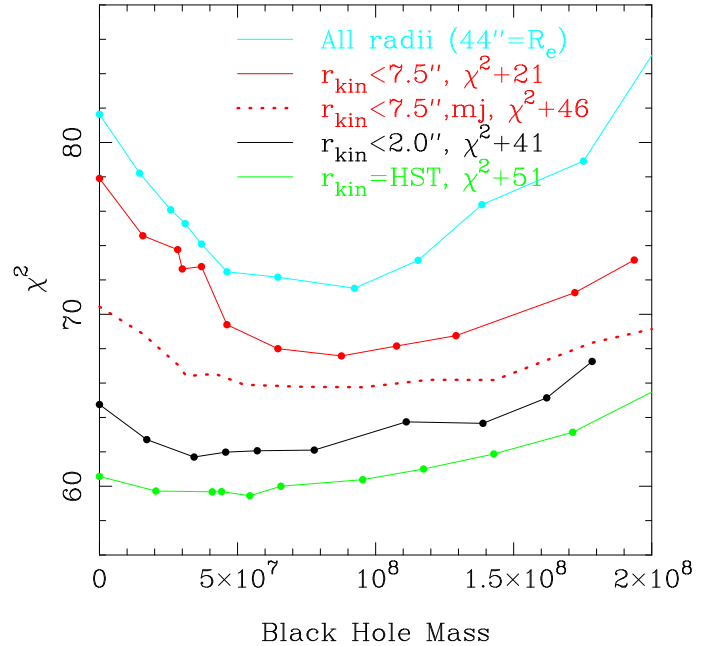


FIG. 2.— Goodness of fit χ^2 versus model black-hole mass in for different sets of observational data from NGC 821. All models were constructed with the 10,000 orbit library described in §2. All of these datasets comprise continuous (i.e. there are no gaps in spatial coverage) long-slit spectra from space- and ground-based observations. The solid lines illustrate the effect (and desirability) of extending the major-axis data to larger radii. The dotted profile shows the effect of removing minor-axis data. If the errors in M_\bullet are determined by the criterion $\Delta\chi^2 = 1$ then all of the profile minima are consistent with $M_\bullet = 8.5 \times 10^7 M_\odot$.

Turning to point 4, sparse radial sampling of the kinematic data (which we never do) can also produce a flat bottomed χ^2 profile [as illustrated in Figure 3 in Richstone & Tremaine (1985)] by pushing up or down velocity moments at unobserved radii to maximize or minimize model parameters.

Kinematic data with better geometry (both resolution and spatial coverage) leads to smaller uncertainties in the BH mass. However, in all of the examples discussed above, the shape of the χ^2 profile provides an accurate estimate of this uncertainty, no matter how poor the data geometry may be. In the absence of

¹² The complete set of observations modeled in Figure 2 consists of STIS spectra and ground-based data obtained at MDM Observatory. The STIS data were observed through the $0''.1$ wide slit with the central pixel centered on the galaxy and the data aggregated in a set of “observations” binned in rectangles with outer edges at $0''.025, 0''.075, 0''.125, 0''.225, 0''.425, 0''.625, 0''.925$. The ground observations were obtained through a $1''.0$ slit binned into rectangles with outer edges at $1''.48, 2''.2, 3''.6, 5''.8, 9''.2, 14''.6, 23''.2, 36''.2, 51''.0$. The minor axis data only extend to $23''.2$.

good spatial coverage, data that resolves the radius of influence, or sufficiently high signal to noise, the BH mass will simply not be well determined, but in our experiments the true mass still lies within our error bounds.

4. DISCUSSION

We have made a point of distinguishing between *imprecise* (the error bars are large) measurements of the BH mass M_\bullet and *unreliable* ones (the error bars are wrong).

Figure 1 shows that increasing the number of orbits beyond our standard prescription does not broaden the range of BH masses allowed by our method. Contrary to the assertion of VME, we must be using enough orbits.

The issue of data sufficiency is far more complex. In §3 we illustrate (or cite) several ways that data insufficiency can lead to imprecise M_\bullet estimates. We also argue that these estimates are *imprecise*, not *unreliable*. Probably the best evidence for this is the fact that our M_\bullet estimates obtained with genuine orbit-superposition models lie within their own errors of repeat studies with greatly improved resolution (Gebhardt *et al.* 2003)¹³.

Flat-bottomed profiles—a special case of imprecision—can be produced by kinematic data with limited radial or angular coverage, by sparsely sampled data, or by insufficient resolution. A fourth way to produce a flat bottomed χ^2 profile is the use of *noiseless* datasets constructed from two-integral models (Cretton & Emsellem 2004). This procedure creates models that match the data perfectly. Since axisymmetric models have more freedom than two-integral models, a range of three integral distribution functions will—by construction—provide an *exact* match to the data. Since χ^2 is bounded by zero, a flat bottom in the χ^2 plot is unavoidable.

Cretton & Emsellem (2004) argue that regularization (smoothing the distribution of orbit weights) reduces the uncertainty in M_\bullet . Our models use a form of regularization (maximum entropy) but only as a numerical technique to accelerate convergence: we iterate our models while steadily reducing the weight of the entropy to the best-fit criterion until there is no further change in the best-fit model. Verolme *et al.* (2002b)

study models with and without regularization for M32 and find no difference in the best fit BH mass.

This paper has examined the two most serious recent criticisms of BH mass estimates from orbit superposition. We conclude that the issues raised by these criticisms have not led us to report erroneous BH masses. There are a number of other tests of our procedure that also indicate that our method is reliable:

1. our method has been successfully blind-tested against (spherical) Jaffe models containing BHs;
2. our method yields the same masses as the Leiden code when applied to the same data (for NGC 821 and M32);
3. BHs weighed by this method display the same distribution of residuals from the mass-velocity dispersion relation as BHs weighed via gas dynamics (Richstone 2004).

The tests described above are posted on the Nuker Team Website (<http://www.noao.edu/noao/staff/lauer/nuker.html>).

The BH masses published by our group (Gebhardt *et al.* 2000a; Bower *et al.* 2001; Gebhardt *et al.* 2003; Siopis *et al.* 2004) are obtained using an adequate number of orbits. The plots of $\chi^2(M_\bullet)$ we publish provide the reader with an accurate assessment of the precision of the results. So far as we can tell, BH masses published from the Leiden group (van der Marel *et al.* 1998; Cretton & van den Bosch 1999; Verolme *et al.* 2002b; Cappellari *et al.* 2002) also should be reliable.

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¹³ We have not investigated filled kinematic maps in radius and angle, as might be obtained with integral-field devices like SAURON. Especially where these data imply triaxiality (thereby violating the basic assumption of our axisymmetric models), there may be unexpected changes in M_\bullet .